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Evolution of longshore beach contour lines determined by the E.O.F. method*

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SUMMARY: Detailed topo-bathymetric levellings were performed biannually for four years at Victoria Beach (Cadiz, Spain) after a beach renourishment carried out in Spring 1991. The subsequent time series were analysed using the Empirical Orthogonal Functions (EOF) method. The evolution of some characteristic longshore contour lines, such as the Highest High Water Level and the Lowest Low Water Level, is studied. The mean coastal line is related to the first spatial EOF mode. Furthermore, an objective criterion for distinguishing between a generalised recession and cyclic accretion-erosion processes due to seasonal sea-swell changes is described, and a uniformly clockwise turn of the shoreline to minimise longshore transport is identified.

Key words: beach changes, longshore variations, Victoria Beach, EOF, Cadiz.

INTRODUCTION

Beaches adapt to different conditions imposed by wind, waves, water levels and current variations through the movement of the sand grains of which they are composed. Detailed topo-bathymetric levellings must be carried out regularly in order to acquire a sufficient amount of data for the thorough understanding of their behaviour in the presence of these events. Among other essential statistical methods used in approaching the laborious study of this data collection, it is only fair to highlight Empirical Orthogonal Functions (EOF) in the coastal engineering field. Although this method has been most assiduously applied in the study of transversal profiles (Winant *et al*., 1975; Aubrey, 1979; Dick and Dalrymple, 1984; Zarillo and Liu, 1988; Pruszak, 1993), it can also be found in application to other types of problems: Losada *et al*. (1991) and Liang and Seymour (1991) applied this technique to analyse the alongshore variations of specific level contour lines; Medina *et al*. (1991) use the EOF method to study transversal transport of sediment; cross-shore distribution of sediment has also been studied by Losada *et al*. (1992) and Medina *et al*. (1994); both cross-shore and alongshore interactions have been analysed by means of the three-mode EOF method (Medina *et al*., 1992); Larson *et al*. (1999) considered short and long term responses of beach fills; etc.

The aim of this paper is to study a temporal series of topo-bathymetric data at Victoria Beach (Cadiz,

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SW Spain) through the application of the Empirical Orthogonal Functions method and to show how this method facilitates the distinction of different longshore variations that take place at the beach. Furthermore, the reconstruction of longshore contour lines based on eigenfunctions appears to be a clearer way of interpreting the results.

STUDY AREA AND FIELD DATA

Victoria is a three km long beach located in the Gulf of Cadiz, facing the Atlantic Ocean on the south-west coast of Spain, near the Strait of Gibraltar (see Fig. 1). Some submerged rocky shoals in front of its shoreline furnishes it with a certain amount of heterogeneity. The beach itself is composed of fine-medium sand (D_{50}=250 μm) consisting of 90-95 percent quartz and 5-10 percent bioclastic material. The region has a mesotidal range with a medium neap to spring variation (1.20-3.30 m). The wave data are recorded through a buoy belonging to the Spanish Network of Wave Registry and Measuring (REMRO) which has been carrying out these operations since 1982, right off the beach shore, at a depth of 22 m. Based on the data collected, analysed and published (MOPT, 1992), the conclusion can be reached that most incident waves originate from the N-W sector, those coming from the East are also frequent and those from the South are less significant. The littoral of Cadiz, embracing the zone between the Guadalquivir River Mouth and the Strait of Gibraltar, has a step-like morphology conditioned by the existence of two groups of fractures, whose tectonic directions are NNW-SSE and E-W (Gutierrez Mas *et al.*, 1991). The coastal stretch under study herein, Victoria Beach, holds this NNW-SSE orientation, allowing northern and western storms to induce a substantial transport of sediments towards the South, whereas the storms proceeding from the East cause the sand to be transported towards the North instead. Nevertheless, the decrease in height of these latter waves allows for the net quantity of sediments –taken to be the algebraic sum of transports caused by each of the individual wave directions– to be from North to South, with an average loss value of no more than 100,000 m^3/year (Muñoz-Perez, 1995a).

A renourishment project of two million m^3 of sand took place in spring 1991. The sand was extended along the 3 km of beachshore with a berm width of a constant 100 m. The berm elevation was of one and a half metres above the maximum high tides. The monitoring project was performed during the following four years to examine the evolution of the fill. Topo-bathymetric levellings were carried out biannually to take the sea-swell seasonality into account.

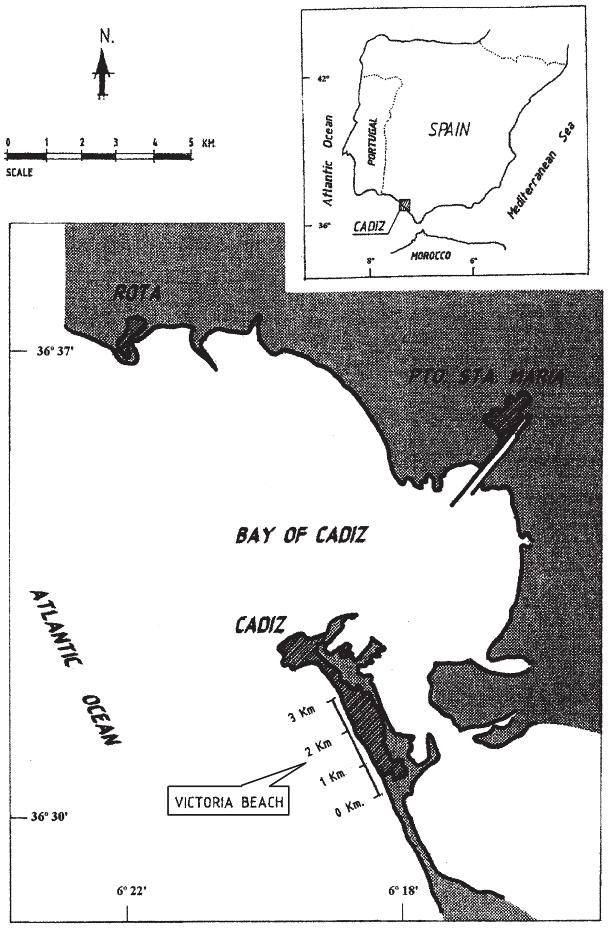

FIG. 1. – Location map of Victoria Beach (Cadiz, SW Spain)

The decision on the execution date was made on the consideration that the months from December to April procure occasional storm activity in the Gulf of Cadiz. So, according to Muñoz-Perez (1995a), two facts were established: the erosion period ends in May and the sand accretion period last until November. For this reason, the topo-bathymetric levellings were put into effect from November-December and from May-June respectively. The first of the seven bathymetries took place in December 1991, while the last was in December 1994. Consequently, the other surveys took place in December 1992, the end of November 1993, June 1992 , the end of May 1993 and in June 1994.

Thirty cross-shore profiles, with a separation between them of one hundred metres, extended from the sea promenade to 6 m depth along the 3 km beach length. This depth is sufficient to cover the changes in beach elevation. According to Hallermeier (1978) and Birkemeier (1985), significant alongshore transport and intense on/offshore transport by waves would be restricted to depths of less than approximately 6 m, being that the nearshore storm wave height that is exceeded only 12 hour/year is about 4 m (MOPT, 1992). Notwithstanding the aforementioned, no substantial changes can be observed beyond the four m depth point through the profile collected over four years of monitoring. Figure 2 shows a specific, yet representative, case of the mean behaviour of said profiles in the area of highest variation incidence. Notice how the maximum slope, of approximately 2%, can be seen in the intertidal zone along with discrepancies in the contour elevation reading of more than a metre.

The bathymetric works were accomplished with sounding equipment loaded onto a vessel and monitored by the Differential Global Position System (DGPS). Each survey campaign lasted at most three days, depending on whether or not both daily high tides could be profitably made use of. The high tide hours were engaged in getting data as close as possible to the shore thanks to the low draft. On the other hand, the topographical work was carried out during long low tide periods, on the same dates. Any decisions about rod stations were left to the rod man due to his/her precise training. This approach made it possible to distinguish those locations where pronounced changes in the seabed topography are to be found and reach the -1.00 m depth. An extended overlapping area from both data collections was covered by using this arrangement. Simultaneous tidal readings were taken with the equipment installed in this area. This all proved valuable for guaranteeing the continuity of the surveyed profiles, as well as providing a common reference point with respect to a bench mark placed up on the promenade.

Aleatory, yet non-systematic errors made in the horizontal DGPS positioning were validly verified as being lower than the two metre range. Therefore, for slopes within the transversal scope of 2%, aleatory errors of up to 4 cm, at most, were induced for this reason in the estimation of depth. Also worthy of notice is that at a maximum speed of 4 knots (2 m/s), the reading is 2 points per second, allowing for one measurement per profile metre. In the measurement of water column height by means of the ecosounder, certain conditioning, such as water temperature and density, affect the accuracy of the sensor. Therefore, calibrating measurements, put into effect at the start and at the end of the daily procedures, were taken through the use of a metal plaque. This plaque was placed at different depths and held by a rigid chain marked with numbered fathoming. The maximum error detected was 4 cm, whereas the average squared error did not exceed 2 cm. For a more detailed description of the accuracy of the method, Muñoz-Perez (1995b) should be consulted.

EIGENFUNCTION ANALYSIS

Empirical Orthogonal Function (EOF), a statistical method also known as Principal Component Analysis (PCA), provides a technique for separating the spatial from the temporal variability of beach profile data. A detailed description of the method can be found in statistics text books (e.g. Daultrey, 1976; Jackson, 1991). In brief, if a function $x = (h_o,y,t)$ represents the adaptation of a longshore contour line to a particular one (h_o), this function may then be defined as a linear combination of scarce spatiality eigenfunctions, e_n (y), and temporal eigenfunctions, c_n (t), by

$$x\,(y_i,\,t_j) = \Sigma\, e_n\,(y_i)\, c_n\,(t_j)\,(\lambda_n\, n_y\, n_t)^{1/2}$$

where x is the offshore direction, n_y is the number of points in the longshore direction, n_t is the number of campaigns carried out to measure the longshore contour line and λ_n is the eigenvalue associated with the n^{th} eigenfunction. The eigenfunctions are ranked according to the percentage of the variability which they explain. This variability is represented by the Mean Squared Value of the data (MSV). Sometimes (Winant *et al.*, 1975), the mean value is of such importance, when explaining the variability, that it must be removed from the original data in order to allow for better and clearer identification of other smaller, yet important, changes. Then, the MSV turns into the Variance. The first eigenfunction explains most of the mean square value of the data, the second eigenfunction explains the greater part of the remaining MSV, and so on. Furthermore, according to Aubrey (1978, 1979), although there is no reason to assume beforehand that the eigenfunctions will represent a physical process, you can observe that the sand is moved in response to wave forcing. One might therefore hope that, since the wave forcing provides most of the variability, the eigenfunctions would reflect it. By examining the spatial and temporal structures of the beach eigenfunctions, one can decide whether to represent some physically meaningful processes.

Moreover, a few properties of the empirical eigenfunctions should be considered (Davis, 1976; Aubrey, 1978):

1.- The EOF provides the most efficient method of compressing data. In other words, the first *n* terms in the EOF expansion represent more of the data variability than the first terms of any other expansion.

2.- Since both the spatial and temporal EOF are orthogonal sets, each corresponding set, $e_n(y_i)\ c_n(t_j)$, is uncorrelated with any other one.

3.- The EOF method provides a means of removing the noise (or less predictable part) from the data.

Eigenfunctions are similar in some aspects to Fourier Analysis. Nevertheless, in Fourier Analysis, data are adjusted to a series of sines and cosines. Conversely, in the EOF, the data itself determine the form of the orthogonal functions.

Supplementary, yet mandatory, the data should be separated into equal gaps both in space and in time, in order to be analysed by the EOF method. This condition is carried out on the time scale, because all seven surveys discussed herein were taken periodically, every six months. The differences in the exact theoretical dates never went beyond two weeks. As for spatial data, certain interpolations were to be made at a distance of ten metres between the different points. Therefore, 61 point series were necessary to cover a transversal distance of 600 metres.

RESULTS AND DISCUSSION

To better understand the behaviour of the beach, some of the more representative longshore contour lines were studied:

- The Highest High Water Level (HHWL), or +4.00 m high contour line, is the maximum spring flood tide line, and approximately the limit between the dry slopeless beach and the beginning of the beach face with a more or less 2% slope.

- The contour line located in the middle of the intertidal zone (+2.00 m), one of the more variable profile points (Muñoz-Pérez *et al.*, 2000).

- The Lowest Low Water Level (0.00 m), coinciding with the spring ebb tide or point at which the submerged beach begins.

- The -2.00 m depth contour line, where considerable changes in the profile are still to be noted (Fig. 2)

The results of these four contour lines obtained in the seven beach levelling studies are presented in Fig. 3. The solid lines show the bathymetric measurements taken at the end of the warm climate season (November-December), i.e., those induced by swell type waves. By contrast, the dashed lines show those taken during May and June, i.e., after the stormy weather or sea waves season. No other aspect can easily and immediately be observed because the graphic type presented is not the most suitable representation of data for showing the evolution of the time series. Consequently, the application of a statistical method was needed. The motivation for selecting the EOF method was based on the characteristic properties explained above.

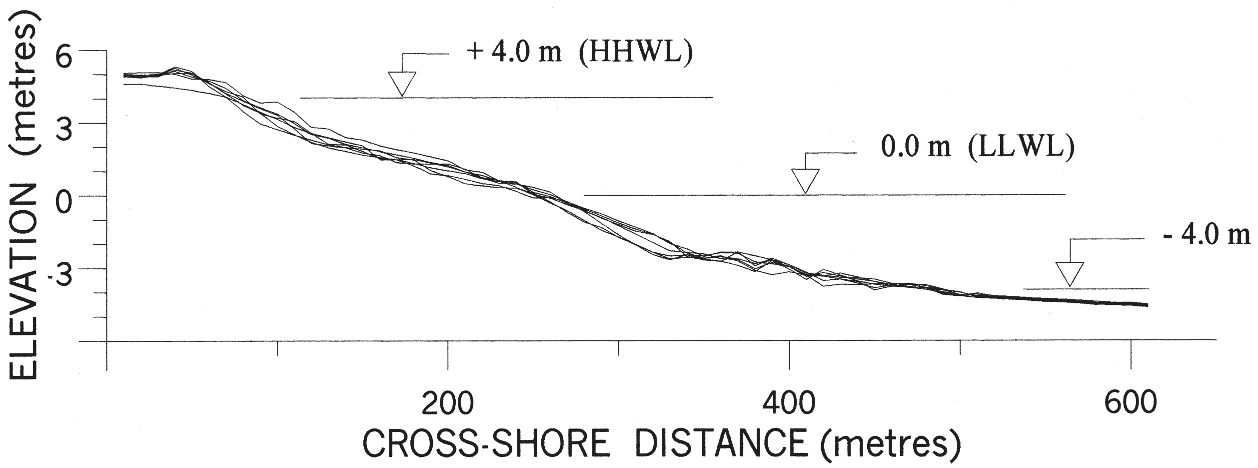

FIG. 2. – Topobathymetric levelling of a representative profile carried out biannually from Dec. '91 to Dec. '94 in Victoria Beach.

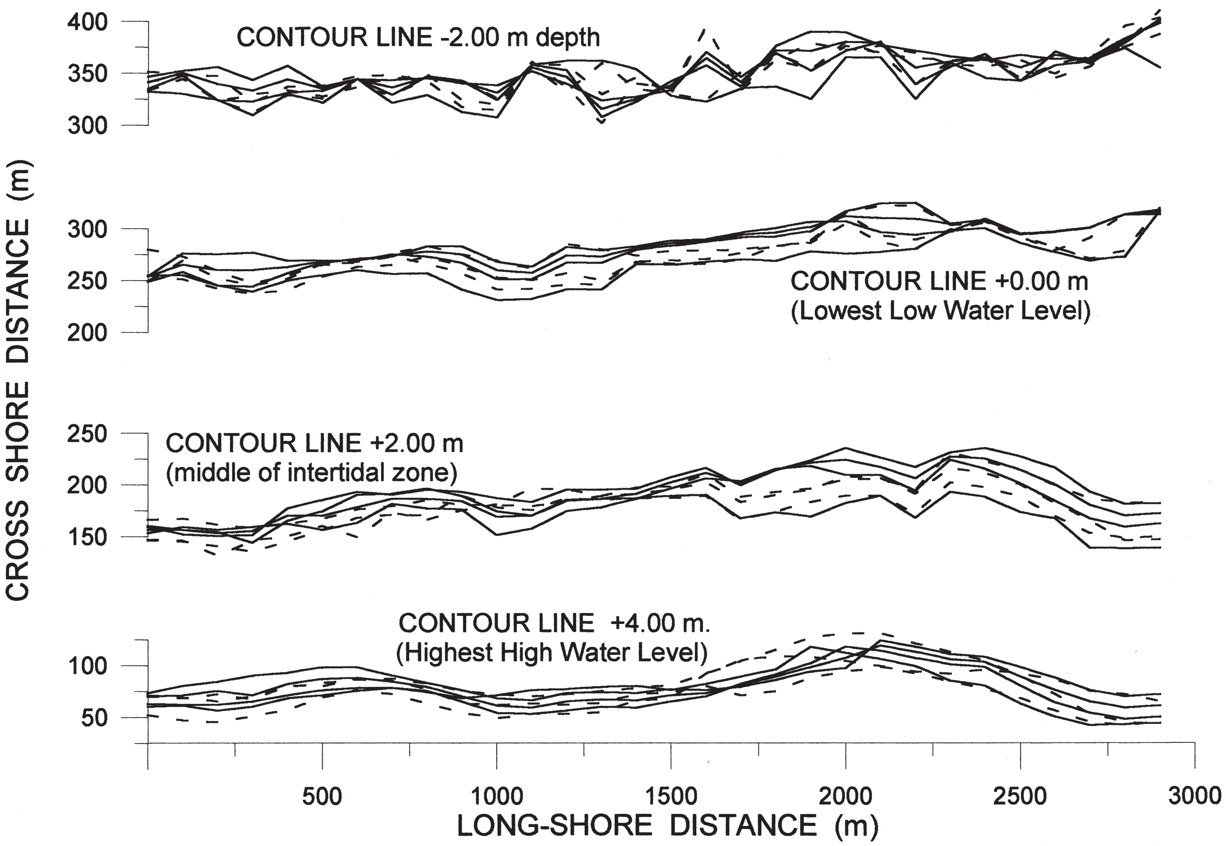

FIG. 3. – Representation of four contour lines (Highest High Water Level, the middle of the intertidal zone, Lowest Low Water Level and -2.00 m depth) obtained in the seven levellings carried out biannually in Victoria Beach from Dec. 1991 to Dec. 1994.

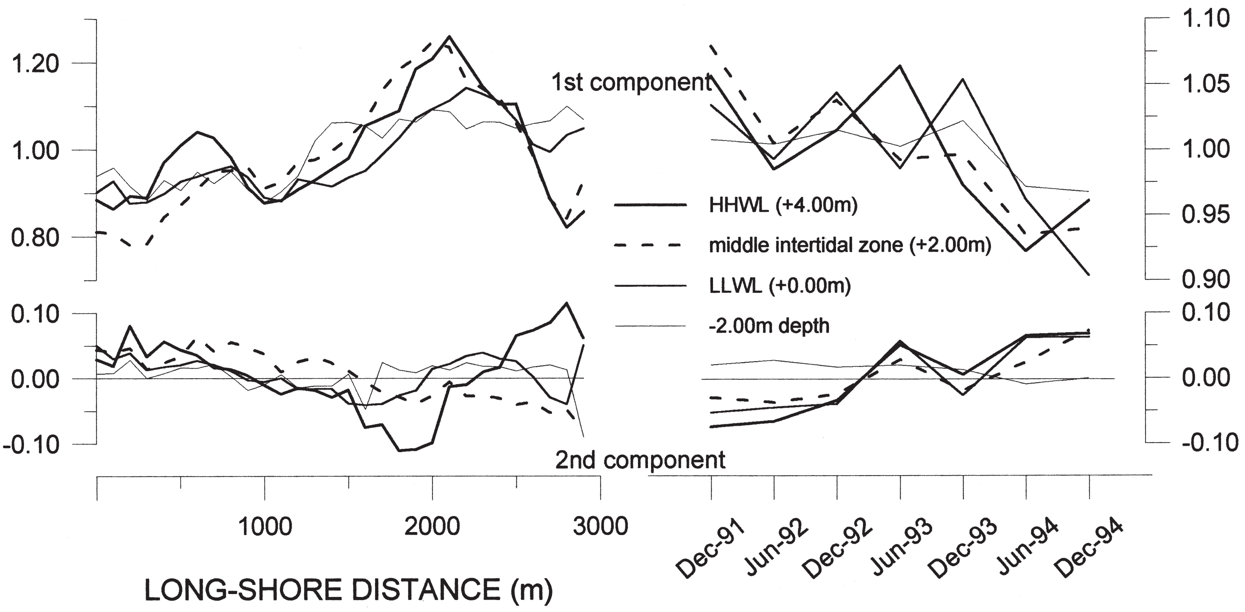

FIG. 4. – First and second spatial and temporal eigenfunctions from the HHWL, middle intertidal zone, LLWL and -2.00 m depth contour lines.

The first and second spatial and temporal eigenfunctions are shown in Fig. 4. The first eigenfunction accounts for between 99.50 and 99.75% of the MSV of the data to the four contour lines (see Table 1), which is very similar to the results found by Dick and Dalrymple (1984) at Bethany Beach (Delaware). Nevertheless, if the mean were removed from the data, as many authors have done (Winant *et*

TABLE 1. – Explanation of the Mean Square Value (MSV) and the Variance (mean removed) of the data by the 1st and 2nd eigenfunctions.

Contour line	MSV Mean included 1st component (%)	 2nd component (%)	Variance Mean removed 2nd component (%)
HHWL (+4.00 m)	99.51	0.32	65
+ 2.00 m height	99.75	0.14	56
LLWL (0.00 m)	99.50	0.26	52
- 2.00 m depth	99.72	0.16	57

al., 1975; Aubrey, 1978, 1979; Dick and Dalrymple, 1984, amongst others), then the second eigenfunction would represent between 52 and 65% of the data Variance (Table 1).

As Losada *et al.* (1991) had previously established, the first spatial eigenfunction represents a mean situation of the longshore contour lines taken during the survey. It is therefore an objective criterion for defining the mean shoreline. The associated temporal eigenfunction shows a biannual seasonality that represents advances and recesses of the coastal line with respect to its median, coinciding with the ends of summer and of winter respectively. Moreover, a decreasing tendency is observed. This implies a net regression of the shoreline as time elapses.

An easier interpretation of the longshore contour line changes and their relation to the forces of nature can be obtained by reconstructing the original data using their respective eigenfunctions. We can thus eliminate all the noisy background accompanying the data output. For the sake of simplification, only the data pertaining to the first (Dec. 1991 and Jun. 1992) and last (Jun. 1994 and Dec. 1994) two bathymetries will be analyzed in this paper. The reconstruction of the HHWL mean coastline (1st eigenfunction) and its discrepancies with the 2nd component are shown in Fig. 5.

Readers are urged to take notice of an interesting point: there exist two different kinds of regression-advance movements of the HHWL contour line parallel to itself. For one of them, the movement that occurred from Jun. 1994 to Dec. 1994 is due to the profile tilting from an upper to lower slope on account of waves being converted from swell to sea and vice-versa (see Fig. 6a). For the other, the movements observed from Dec. 1991 to Jun. 1992 (Fig. 6a) and from Dec. 1991 to Dec. 1994 (or from Jun. 1992 to Jun. 1994) correspond to the generalized regression caused by the loss of 300,000 m^3 of sand during those years (Muñoz-Perez, 1995a). With time, the evolution of sediment volume obtained through the comparison of the succeeding levelling series is illustrated in Fig. 7. Note that, as sand loss

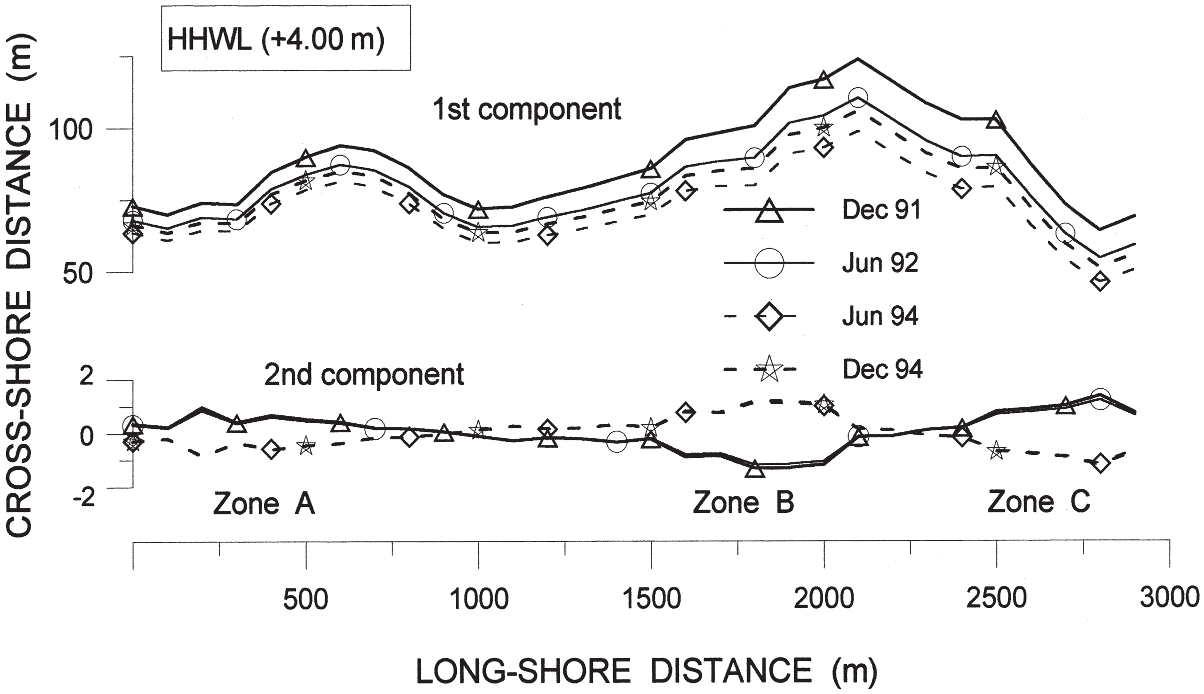

FIG. 5. – Reconstruction of HHWL contour line using the 1st eigenfunction. Differences attributed to the 2nd component are also shown.

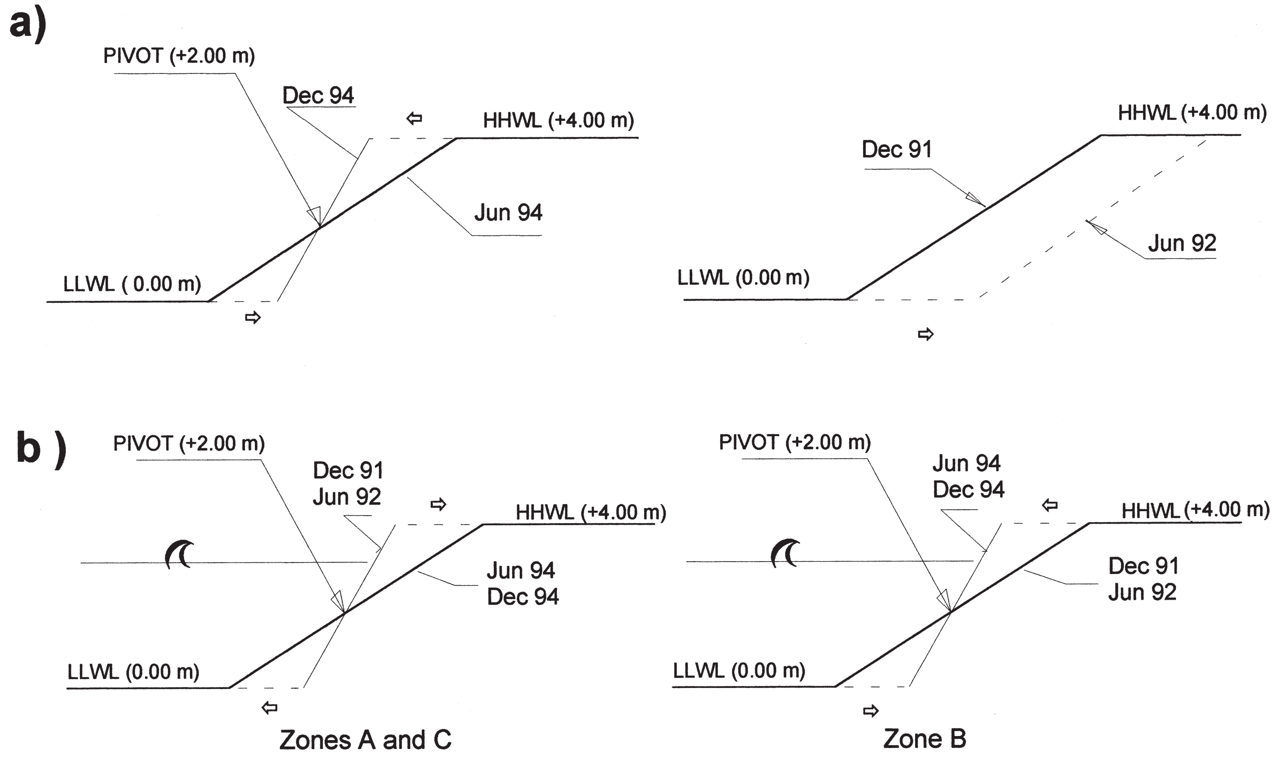

FIG. 6. – Rough sketch of changes occurring in the beach profile as an interpretation of the 1st (5a) and 2nd (5b) eigenfunctions. a) tilting due to seasonal sea-swell changes from Jun. '94 to Dec. '94 and first general regression from Dec. 1991 to Jun. 1992; b) tilting due to adaptation of beach fill to the original boundary conditions in the different beach zones (A-C and B) from Dec. 1991 to Dec. 1994.

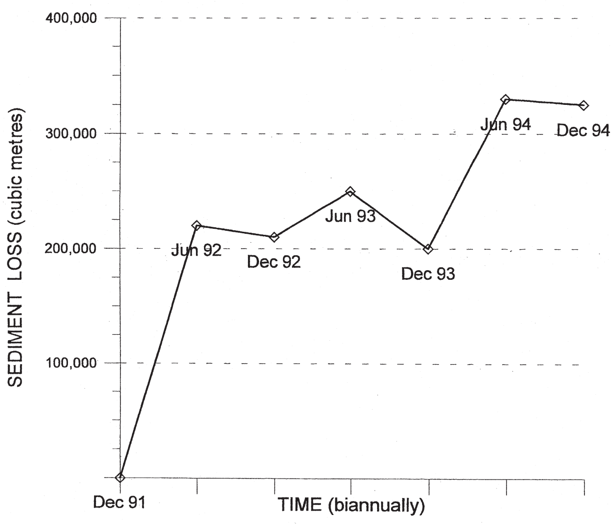

FIG. 7. – Loss of sediment obtained through comparison between the different levellings.

is of such importance during the first six months (2/3 of the total), the seasonal tilting process of the profile from Dec. 1991 to Jun. 1992 remains camouflaged.

The overlapping of line Dec. 1991 with Jun. 1992 and Jun. 1994 with Dec. 1994 can clearly be observed in the contribution of the second component. This implies the disappearance of any differences through the alternation sea versus swell waves. Great attention must be paid to the division of our beach into three zones. At both ends of the beach (A and C) there is a well defined regression of the shoreline over the course of the years, while the centre (B) advances towards the ocean. The interpretation of this process must be integrated with the LLWL.

The midway point contour line of the intertidal zone shows its evolution in Fig. 8. A generalised regression is observed which decreases down to total nullity (Jun. 1994 coinciding with Dec. 1994). This indicates that there is not an advance-regression phenomenon due to the alternating sea-swell effect and, therefore, this contour line appears as a pivot line (Fig. 6). The second component indicates a dextrogyrous turn of the shoreline of approximately 2°. We venture to say that the objective is the adaptation of the shoreline to the wave direction and minimisation of longshore transport. The fact that the net transport direction is from North to South and that a clockwise

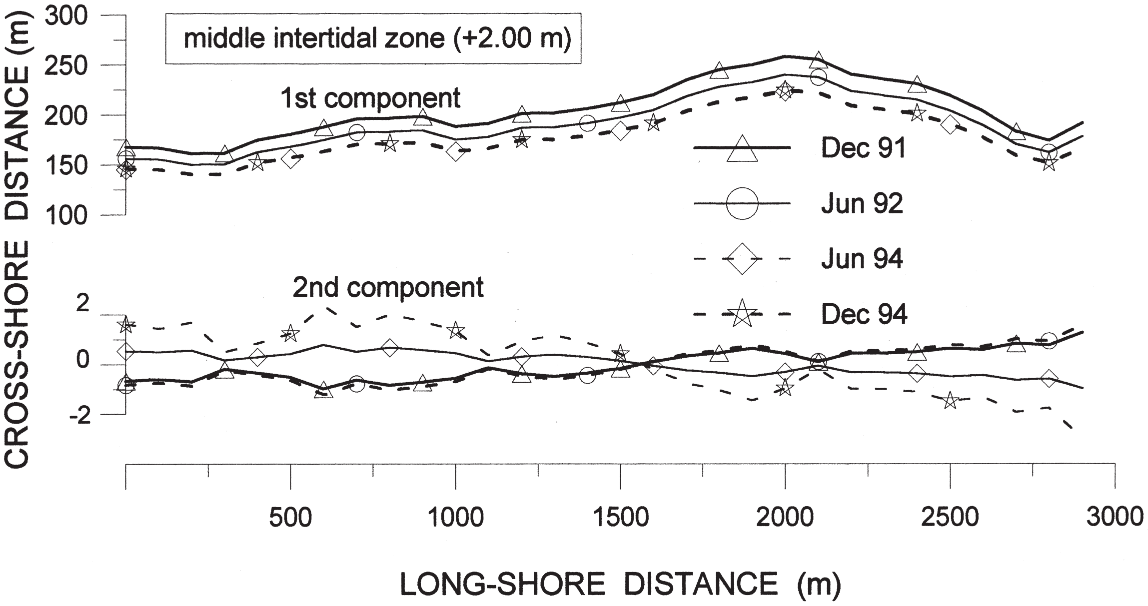

FIG. 8. – Reconstruction of the middle intertidal contour line using the 1st eigenfunction. Differences accrued to the 2nd component are also shown.

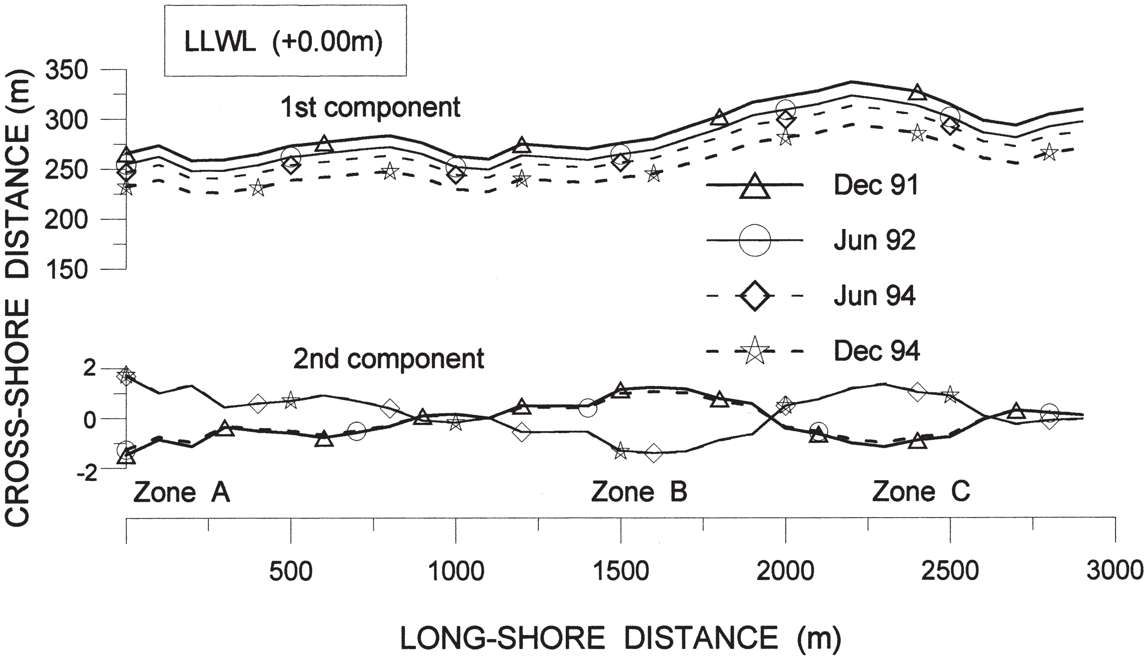

FIG. 9. – Reconstruction of the LLWL contour line using the 1st eigenfunction. Differences due to the 2nd component are also shown.

rotation implies a shore angle decrease with respect to the breaking line of waves originating from the WNW is worthy of consideration.

The LLWL contour line is shown in Figure 9. A generalised regression is appreciated to be similar in nature to the other two level curves. The difference with respect to HHWL originates in the fact that in Dec. '94, it appears closer to land than in Jun. '94. This behaviour is coherent with the previous interpretation of the seasonal tilting in the profile (Fig. 6). Moreover, the second component shows the very same three zones observed in the HHWL, but, contrarily, the LLWL advances towards the sea in the outer areas (A and C), whereas it recedes in the central zone (B). This evidence, coupled with the assumption that the middle point of the intertidal zone is a pivotal point, let assure that profiles have become milder at the ends of the beach, whilst they

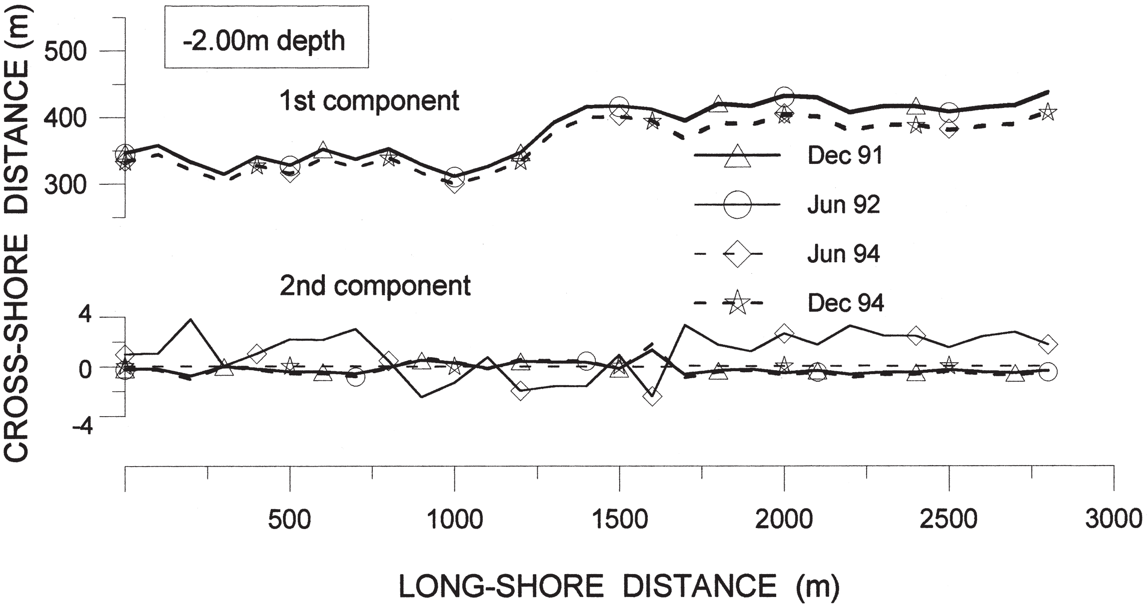

FIG. 10. – Reconstruction of the -2.00 m depth contour line using the 1st eigenfunction. Differences due to the 2nd component are also shown.

have become more abrupt in the central zone (Fig. 6b). This behaviour may be attributed to the presence of rocky shoals, the consequent non-homogeneity of the beach and the adaptation of the renourished profiles to the original boundary conditions.

As with the other cases, a generalised regression is also appreciated in the 2.00 m depth line (see Fig. 10), but no seasonal movements are observed given that Jun. 1992 and Jun. 1994 coincide with Dec. 1991 and Dec. 1994 respectively. It may be inferred that the profile tilting does not reach this depth. Regrettably, the second eigenfunction could not be given any physical interpretation in this case.

Finally, a direct measurement of the reconstructed shorelines allowed us to establish an averaged generalised regression of 26 m, ranking from 20 to 32 m, while the seasonable sea-swell changes entail quasi-constant oscillations of only 5 m.

CONCLUSIONS

The longitudinal development of Victoria Beach after its regeneration in 1991, as well as the topo-bathymetric monitoring of the four years post hoc, were analysed using Empirical Orthogonal Functions. This method has proved to be a very useful statistical instrument for showing and analysing the changes that occur in different shorelines and for finding the physical mechanisms to which they are related.

The reconstruction method of contour lines by exclusive use of the first eigenfunction allowed an objective criterion for defining the shoreline to be established. Furthermore, given that the middle of the intertidal zone appears as a pivotal point, this contour line may be assumed as an equilibrium line that is not affected by the seasonal changes. Conversely, the changes that occur in the presence of the sea-swell alternations, both in the HHWL and in the LLWL, are included and may be surveyed. The HHWL migrates seaward during the summer and landward during the stormy period, and LLWL moves in an opposite fashion creating a more abrupt or milder slope respectively. So, this phenomenon may be distinguished and separated from the generalised regression.

The second component shows how the beach fill adapts to the original boundary conditions. For one, two tiltings of the transverse profile into different directions are detected in the HHWL and LLWL lines according to the beach zones. The extremes tend to soften the profile, whereas the slope increases in the centre. For another, a clockwise twist of the shoreline can be perceived in the contour line located in the middle of the intertidal zone.

Finally, the first component of the submerged contour line (-2.00 m) has only undergone a generalised regression, lacking the usual alternations of the intertidal zone, which could mean that these floors are not affected by the seasonal tilt of the profile.

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